A Stationary Electromagnetic Toroidal Structure

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Abstract

Electromagnetic fields are usually associated with waves that propagate through space and transport energy from one location to another. This paper explores a different possibility: an electromagnetic field organized into a toroidal structure that remains at the same location while its internal fields continue to oscillate.

The proposed structure consists of electric and magnetic fields that oscillate with a quarter-period phase difference. As a result, energy is periodically exchanged between the electric and magnetic fields. The associated Poynting flow also oscillates and repeatedly reverses direction, giving no persistent net transport of energy in one direction. The structure can therefore remain stationary as a whole while being continuously dynamic internally.

The electric and magnetic fields also produce opposing electromagnetic stresses. Rather than maintaining a perfectly rigid shape, the torus is therefore expected to expand and contract periodically. Its stationary state is interpreted as a dynamic equilibrium in which field oscillation, internal energy flow, and geometrical deformation form a repeating cycle.

A particular energy-balanced configuration exists when the maximum electric and magnetic field energies are equal. Other field amplitudes may also be possible if changes in the toroidal geometry compensate for the imbalance. The principal unresolved question is whether such a finite-energy, stable, non-radiating and self-confined toroidal field configuration can exist as a complete solution in free space.

If such a configuration exists, it could represent a simple form of localized electromagnetic organization and a possible starting point for considering more complex structures of matter.

1. Introduction

Electromagnetic fields are commonly described in terms of propagating waves, in which electric and magnetic field components travel through space while continuously transporting energy. Maxwell's equations, however, also permit standing and resonant field configurations in which the local electromagnetic fields vary periodically while the time-averaged transport of energy can vanish. This raises a more fundamental question: **can an electromagnetic field form a localized toroidal structure whose centre remains stationary while its internal fields continue to oscillate?**

A toroidal geometry provides an interesting framework for considering such a possibility. In the model explored here, the magnetic field is predominantly directed along the toroidal path, whereas the electric field is predominantly transverse to this direction. The electric and magnetic fields are assumed to oscillate with an approximate phase difference of one quarter of a period. A representative local description is therefore

$$E(t) = E_A \cos \omega t,$$
$$B(t) = B_A \sin \omega t.$$

Such a phase relation has an important consequence. Since electromagnetic field energies depend quadratically on the field amplitudes, the electric and magnetic energy components vary as

$$W_E \propto \cos^2 \omega t, \qquad W_B \propto \sin^2 \omega t.$$

Under an energy-balanced condition, the decrease of one component can therefore be accompanied by a corresponding increase of the other. The electromagnetic structure need not be static internally even when its macroscopic position remains unchanged.

The same phase relation also produces an oscillating Poynting vector. Since

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B},$$

the corresponding energy flux varies as

$$S \propto \sin(2\omega t).$$

Its direction consequently reverses periodically and its time average over a complete oscillation is zero. This distinguishes the proposed configuration from a propagating electromagnetic wave: energy may be continuously redistributed within the structure without producing a persistent net energy flow in one spatial direction.

A second essential aspect of the model is that the toroidal geometry itself is not assumed to be rigid. Electric and magnetic fields produce stresses described by the Maxwell stress tensor. In the proposed configuration, the electric contribution tends to expand particular dimensions of the torus, whereas magnetic tension associated with curved magnetic field lines can produce an opposing contraction. Since the electric and magnetic energies oscillate, these stresses also vary periodically. The torus is therefore expected to undergo a periodic geometrical deformation rather than remain at a fixed size.

To describe this dynamics, two characteristic dimensions are introduced: the major toroidal radius R and the minor radius a of its cross-section. Both may vary with time,

$$R = R(t), \qquad a = a(t).$$

The stationary character of the structure should therefore not be understood as absence of internal motion. It refers only to the absence of translational propagation of the structure as a whole. Internally, field energy, electromagnetic momentum, and geometry may all undergo periodic variations.

An important distinction will be made between the existence of such an oscillating configuration and its energetically optimal state. Particular amplitudes E_0 and B_0 may provide an optimal balance between the electric and magnetic contributions, but this does not necessarily imply that oscillation is possible only at these amplitudes. A departure from the optimal field ratio may instead be accompanied by changes in the dimensions, frequency, or amplitude of the toroidal oscillation.

The elementary toroidal structure considered in this work is assumed to carry **zero net electric charge**. Consequently, the total electric flux through a closed surface surrounding it must satisfy

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = 0.$$

The possible formation of larger structures from multiple elementary toroidal units, as well as the possible emergence of net electric charge in such composite configurations, lies outside the scope of the present work.

The purpose of this paper is therefore limited and specific. We investigate whether the field oscillations, energy exchange, internal Poynting flow, and electromagnetic stresses associated with a stationary toroidal configuration can be described consistently within an electromagnetic framework. Particular attention is given to the periodic exchange between electric and magnetic field energies, the role of displacement current, the reversal of the Poynting vector, and the resulting oscillation of the toroidal geometry.

The existence of a freely localized and self-confined electromagnetic torus in vacuum is **not assumed as an established consequence of Maxwell's equations**. Rather, it is the central physical hypothesis examined in this work. If a dynamically stable configuration of this type can exist, it would represent an unusual form of electromagnetic organization: a structure that is stationary as a whole while remaining permanently dynamic internally. The possible interpretation of such a structure as a primitive building block of matter is considered only as a broader motivation and is not required for the analysis developed here

2. Geometry of the Stationary Electromagnetic Torus

The stationary electromagnetic structure considered in this work is assumed to have a toroidal geometry. Two characteristic dimensions are used to describe its shape: the major radius R, measured from the centre of the torus to the centre of its circular cross-section, and the minor radius a, representing the radius of that cross-section. For an ideal torus with a circular cross-section, the volume is

$$V = 2\pi^2 R a^2. \qquad (1)$$

Neither R nor a is assumed to remain strictly constant. As will be discussed later, electromagnetic stresses may cause both dimensions to oscillate in time. The quantities R and a should therefore be regarded as equilibrium or instantaneous geometrical parameters, depending on the context.

The local geometry of the torus can conveniently be described by three mutually perpendicular directions. The toroidal direction ϕ follows the major circular path around the torus. The poloidal direction θ follows the circular cross-section of the torus, while the local normal direction n points outward from the toroidal surface.

The magnetic field is assumed to be predominantly oriented along the toroidal direction,

$$\mathbf{B} = B_\phi \, \hat{\boldsymbol{\phi}}. \qquad (2)$$

The magnetic field lines therefore form closed paths following the length of the torus. Their curvature introduces an important geometrical property: magnetic tension along these curved field lines can produce a mechanical effect directed toward the centre of curvature. This effect will later be considered as one of the contributions to the dynamic confinement of the structure.

The electric field is assumed to be predominantly transverse to the magnetic field and locally approximately normal to the toroidal surface,

$$\mathbf{E} = E_n \, \hat{\mathbf{n}}. \qquad (3)$$

A useful geometrical picture is therefore that of a torus surrounded locally by electric-field lines resembling the spines of a hedgehog, while the magnetic field runs along the toroidal body. This picture is intended to specify the local orientation of the fields rather than to imply a net outward electric flux.

The elementary torus considered here is electrically neutral. Consequently,

$$Q = 0,$$

and Gauss's law requires

$$\boxed{\oint_S \mathbf{E} \cdot d\mathbf{A} = 0.} \tag{4}$$

Thus, although the electric field may locally be normal to the toroidal surface, its normal component cannot have the same sign over the entire closed surface. Regions of positive and negative electric flux must compensate so that the total flux vanishes. The detailed spatial distribution $E_n(\theta,\phi)$ is not specified at this stage; only the requirement of zero total flux is imposed.

This distinction between **local field orientation** and **global electric flux** is essential. The simplified local representation in Eq. (3) will be used to analyse electromagnetic energy flow and stress, while Eq. (4) remains a global constraint on any physically admissible field distribution.

2.1. Local orientation of the Poynting vector

The assumed geometry immediately determines the local direction of electromagnetic energy transport. The Poynting vector is

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}. \tag{5}$$

For the field orientations given by Eqs. (2) and (3),

$$\mathbf{E} \perp \mathbf{B},$$

and therefore

$$\mathbf{S} = \frac{E_n B_\phi}{\mu_0} \hat{\boldsymbol{\theta}}, \tag{6}$$

up to the chosen orientation of the local coordinate system.

The energy flux is consequently directed predominantly along the **poloidal coordinate**, i.e. around the cross-section of the torus, rather than along its major circumference.

This geometrical result will become particularly important when the time dependence of the fields is introduced. If E_n and B_ϕ oscillate with a relative phase shift, the sign of their product changes periodically. The local Poynting vector can therefore reverse its direction along the same spatial coordinate,

$$+\hat{\boldsymbol{\theta}} \quad \longleftrightarrow \quad -\hat{\boldsymbol{\theta}},$$

without requiring translational motion of the toroidal structure itself.

2.2. Two geometrical degrees of freedom

The distinction between R and a is also important dynamically. A change in R modifies primarily the curvature of the toroidal magnetic-field lines, whereas a change in a modifies the cross-sectional geometry and the characteristic spatial scale over which the fields vary.

The torus should therefore not be treated as a rigid body characterized by a single radius. Its geometry is more generally described by

$$R = R(t), \qquad a = a(t). \tag{7}$$

Its instantaneous volume is consequently

$$V(t) = 2\pi^2 R(t) a^2(t). \tag{8}$$

For small variations around an equilibrium geometry (R_0, a_0),

$$R(t) = R_0 + \delta R(t), \qquad a(t) = a_0 + \delta a(t),$$

and the relative volume variation to first order is

$$\boxed{\frac{\delta V}{V_0} = \frac{\delta R}{R_0} + 2\frac{\delta a}{a_0}.} \tag{9}$$

Equation (9) allows two qualitatively different forms of geometrical oscillation. The torus may expand and contract as a whole, or its major and minor radii may vary in opposite directions. In the latter case, an approximately volume-preserving deformation satisfies

$$\frac{\delta R}{R_0} + 2\frac{\delta a}{a_0} \simeq 0. \tag{10}$$

The torus would then alternate between a larger and thinner configuration and a smaller and thicker configuration while undergoing only a small first-order change in volume.

2.3. Geometrical model and physical hypothesis

The geometry introduced above does not by itself establish the existence of a self-confined electromagnetic torus. Equations (1) – (10) define the geometrical framework within which the field dynamics will be examined. A complete solution would ultimately require spatial field functions

$$\mathbf{E}(\mathbf{r}, t), \qquad \mathbf{B}(\mathbf{r}, t)$$

that simultaneously satisfy Maxwell's equations, the zero-flux condition of Eq. (4), finite total field energy, and the required boundary behaviour in the surrounding space.

The purpose of the present geometrical model is therefore more specific. It provides a framework for examining whether the assumed orientations of the electric and magnetic fields permit **periodic internal energy transport and opposing electromagnetic stresses without translational propagation of the structure**.

With the geometry established, the next step is to examine the temporal behaviour of the two fields and, in particular, the possibility of a quarter-period phase difference between E and B.

3. Temporal Oscillation of the Electric and Magnetic Fields

The stationary toroidal structure considered here is assumed to remain fixed in its mean spatial position while its electromagnetic fields vary periodically in time. The simplest local representation is

$$E(t) = E_A \cos \omega t, \tag{11}$$

$$B(t) = B_A \sin \omega t. \tag{12}$$

The electric and magnetic fields are therefore shifted in phase by one quarter of a period,

$$\Delta\varphi = \frac{\pi}{2}.$$

This phase relation differs from that of a freely propagating plane electromagnetic wave in vacuum, where the electric and magnetic fields are locally in phase. It is, however, characteristic of standing or resonant electromagnetic configurations, in which electric and magnetic field energies can be exchanged periodically.

The quantities E_A and B_A denote the actual field amplitudes of a particular toroidal configuration. They should be distinguished from the amplitudes E_0 and B_0, introduced later as the amplitudes associated with an energetically optimal state.

3.1. Compatibility with Maxwell's equations

A quarter-period phase shift between E and B does not arise from time dependence alone. It must be accompanied by an appropriate spatial variation of the fields.

This can be seen most simply in a local one-dimensional approximation. Let

$$\mathbf{E}(x,t) = E_0 \cos(kx) \cos(\omega t)\, \hat{\mathbf{y}}, \tag{13}$$

and

$$\mathbf{B}(x,t) = B_0 \sin(kx) \sin(\omega t)\, \hat{\mathbf{z}}. \tag{14}$$

Faraday's law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{15}$$

requires

$$kE_0 = \omega B_0. \tag{16}$$

Ampère-Maxwell's law,

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{17}$$

requires

$$kB_0 = \frac{\omega}{c^2} E_0. \tag{18}$$

Combining Eqs. (16) and (18) gives

$$\boxed{\omega = ck} \tag{19}$$

and therefore

$$\boxed{E_0 = cB_0.} \tag{20}$$

Thus a spatially standing electromagnetic configuration can have electric and magnetic fields in temporal quadrature while remaining fully compatible with the local vacuum Maxwell equations.

The toroidal structure considered in this work is more complicated than the one-dimensional example, but the example demonstrates the important principle: a stationary field pattern does not require static fields.

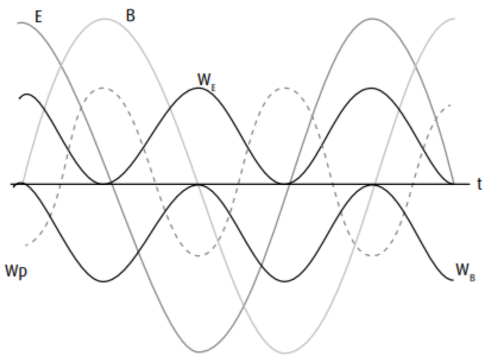

Figure 1. Temporal relations between the electric and magnetic fields and the associated energy exchange.
The electric and magnetic fields are shown in temporal quadrature. The curves WEW_EWE and WBW_BWB represent variations of the electric and magnetic field energies relative to their mean values; their negative portions therefore do not represent negative absolute electromagnetic energy. The quantity Wp schematically indicates the intermediate energy-transfer phase associated with the displacement current and should not be interpreted as an additional independent store of energy.

3.2. Spatial and temporal phase relations

Equations (13) and (14) also show that the electric and magnetic fields are shifted not only in time but also in space.

The electric field has the spatial dependence

$$E \propto \cos(kx),$$

whereas the magnetic field has

$$B \propto \sin(kx).$$

Thus their spatial maxima are separated by a quarter wavelength.

At an electric-field antinode,

$$|E| = E_{\max}, \qquad B = 0,$$

whereas at a magnetic-field antinode,

$$E = 0, \qquad |B| = B_{\max}.$$

The field structure can therefore be interpreted as a periodic transfer between predominantly electric and predominantly magnetic configurations.

In the toroidal geometry, the coordinate x of the simple example is replaced by an appropriate local coordinate following the internal field structure. The exact spatial functions will depend on the toroidal eigenmode and are not specified in the present approximate model.

3.3. Periodicity without translational propagation

For a propagating plane wave, a constant phase condition,

$$kx - \omega t = \text{const},$$

moves through space with velocity

$$v = \frac{\omega}{k} = c.$$

In contrast, the standing form of Eq. (13),

$$E(x,t) = E_0 \cos(kx) \cos \omega t,$$

contains fixed spatial nodes,

$$\cos(kx) = 0,$$

whose positions do not change with time.

The field therefore oscillates, but the spatial pattern itself does not propagate.

This distinction is essential for the present model. The term stationary torus does not imply

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{\partial \mathbf{B}}{\partial t} = 0.$$

Rather, it means that the mean spatial structure remains localized while

$$\frac{\partial \mathbf{E}}{\partial t} \neq 0, \qquad \frac{\partial \mathbf{B}}{\partial t} \neq 0.$$

The internal electromagnetic dynamics may therefore be rapid even though the centre of the toroidal structure remains at rest.

3.4. Actual and optimal amplitudes

It is useful at this point to distinguish two concepts.

The actual amplitudes

$$E_A, \qquad B_A$$

describe a particular oscillating torus.

The optimal amplitudes

$$E_0, \qquad B_0$$

describe a special configuration in which the electric and magnetic contributions satisfy the energy-balance condition discussed in the following section.

In the simplest symmetric case this condition becomes

$$E_0 = cB_0.$$

However, Eq. (21) should not yet be interpreted as a necessary condition for the existence of every toroidal oscillation. More generally,

$$E_A \neq cB_A$$

may describe a non-optimal configuration whose geometry, field distribution, or oscillation frequency differs from the energy-balanced state.

This distinction is important because the field amplitudes and the toroidal geometry are dynamically coupled. A departure from the optimal field ratio may therefore lead to a change in

$$R, \qquad a, \qquad \omega$$

rather than to immediate destruction of the oscillating structure.

3.5. Characteristic frequency and toroidal dimensions

The simple standing-wave relation

$$\omega = ck$$

suggests that the characteristic frequency is linked to the spatial scale of the field distribution.

For a toroidal eigenmode, one may write generally

$$k_{\text{eff}} L_{\text{eff}} = \Gamma,$$

where L_{eff} is a characteristic geometrical length and Γ is a dimensionless eigenvalue determined by the field geometry.

Thus

$$\boxed{\omega = \Gamma \frac{c}{L_{\text{eff}}}.} \tag{22}$$

Depending on the particular mode, L_{eff} may be associated predominantly with the minor radius a, the major radius R, or a combination of both.

This relation establishes an important coupling between geometry and electromagnetic oscillation. A change in the toroidal dimensions can modify the oscillation frequency, while the field stresses generated by the oscillation can in turn modify the geometry.

The stationary torus must therefore be regarded as a coupled system,

$$\boxed{\mathbf{E}(t), \mathbf{B}(t) \longleftrightarrow R(t), a(t).}$$

The next section examines the first consequence of this periodic field behaviour: the exchange between electric and magnetic field energies.

4. Periodic Exchange of Electric and Magnetic Energy

The quarter-period phase relation between the electric and magnetic fields has a direct consequence for the distribution of electromagnetic energy within the stationary torus. Although the field amplitudes oscillate with angular frequency ω, the corresponding field energies depend on the squares of the field strengths and therefore vary with angular frequency 2ω.

The local electric-field energy density is

$$u_E(t) = \frac{\varepsilon_0}{2} E^2(t), \tag{23}$$

whereas the magnetic-field energy density is

$$u_B(t) = \frac{1}{2\mu_0} B^2(t). \tag{24}$$

Using

$$E(t) = E_A \cos \omega t, \qquad B(t) = B_A \sin \omega t,$$

gives

$$u_E(t) = \frac{\varepsilon_0 E_A^2}{2} \cos^2 \omega t, \tag{25}$$

and

$$u_B(t) = \frac{B_A^2}{2\mu_0} \sin^2 \omega t. \tag{26}$$

Thus the electric and magnetic energy components reach their maxima at different times.

4.1. Energy-balanced oscillation

Define the maximum electric and magnetic energy densities as

$$u_{E0} = \frac{\varepsilon_0 E_A^2}{2}, \qquad u_{B0} = \frac{B_A^2}{2\mu_0}.$$

Then

$$u_E = u_{E0} \cos^2 \omega t, \tag{27}$$

$$u_B = u_{B0} \sin^2 \omega t. \tag{28}$$

A particularly simple case occurs when the maximum electric and magnetic energy densities are equal,

$$u_{E0} = u_{B0} = u_0. \tag{29}$$

For identical geometrical weighting of the two field components, this condition is equivalent to

$$\boxed{E_0 = cB_0.} \tag{30}$$

Equations (27) and (28) then become

$$u_E = u_0 \cos^2 \omega t,$$

$$u_B = u_0 \sin^2 \omega t.$$

Since

$$\cos^2 \omega t + \sin^2 \omega t = 1,$$

their sum is constant:

$$\boxed{u_E(t) + u_B(t) = u_0.} \tag{31}$$

In this idealized limit, electromagnetic energy is therefore not periodically created and destroyed. Instead, it is redistributed between electric and magnetic field configurations.

At

$$\omega t = 0,$$

the energy is entirely electric:

$$u_E = u_0, \qquad u_B = 0.$$

One quarter of a field period later,

$$\omega t = \frac{\pi}{2},$$

the situation is reversed:

$$u_E = 0, \qquad u_B = u_0.$$

After another quarter period, the electric energy again reaches its maximum, although the direction of the electric field has reversed.

The energy cycle can therefore be represented schematically as

$$\boxed{W_E \longrightarrow W_B \longrightarrow W_E \longrightarrow W_B \dots} \tag{32}$$

where reversal of the field direction does not reverse the sign of its stored energy.

4.2. Energy oscillation occurs at twice the field frequency

Using

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

and

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x),$$

Eqs. (27) and (28) can be written as

$$u_E(t) = \frac{u_0}{2} + \frac{u_0}{2} \cos 2\omega t, \tag{33}$$

$$u_B(t) = \frac{u_0}{2} - \frac{u_0}{2} \cos 2\omega t. \tag{34}$$

The mean values are therefore

$$\langle u_E \rangle = \langle u_B \rangle = \frac{u_0}{2},$$

while the departures from their mean values satisfy

$$\Delta u_E = \frac{u_0}{2} \cos 2\omega t, \tag{35}$$

$$\Delta u_B = -\frac{u_0}{2} \cos 2\omega t. \tag{36}$$

Consequently,

$$\boxed{\Delta u_B = -\Delta u_E.} \tag{37}$$

This representation is useful for interpreting diagrams in which the electric and magnetic energy contributions are shown as positive and negative oscillating quantities. The negative portion of such a curve should not be interpreted as negative electromagnetic energy. It represents an energy density **below its mean value**.

The physical densities themselves remain non-negative at all times:

$$u_E \geq 0, \qquad u_B \geq 0.$$

4.3. Global energies of the torus

For a spatially extended toroidal structure, the relevant quantities are the integrated field energies,

$$W_E(t) = \int_V \frac{\varepsilon_0}{2} E^2(\mathbf{r}, t)\, dV, \tag{38}$$

and

$$W_B(t) = \int_V \frac{1}{2\mu_0} B^2(\mathbf{r}, t)\, dV. \tag{39}$$

If the spatial profiles of the two fields remain approximately fixed during one electromagnetic cycle, these can be written in the form

$$W_E(t) = W_{E0} \cos^2 \omega t, \tag{40}$$

$$W_B(t) = W_{B0} \sin^2 \omega t. \tag{41}$$

For an energy-balanced torus,

$$W_{E0} = W_{B0} = W_0; \tag{42}$$

and therefore

$$\boxed{W_E(t) + W_B(t) = W_0.} \tag{43}$$

The equality of the maximum integrated energies is slightly more general than the local condition $E_0 = c.B_0$, because the electric and magnetic fields may have different spatial distributions. In general,

$$\frac{\varepsilon_0}{2} \int_V E_0^2(\mathbf{r})\, dV = \frac{1}{2\mu_0} \int_V B_0^2(\mathbf{r})\, dV. \tag{44}$$

Equation (44), rather than a pointwise equality between E and c.B, is therefore the more fundamental energy-balance condition for the complete toroidal structure.

This distinction may become important when an exact toroidal field solution is considered.

4.4. Non-optimal amplitudes

The energy-balanced case represents a particularly symmetric state, but the present model does not assume that every oscillating torus must satisfy Eq. (42).

For arbitrary actual amplitudes,

$$W_{E0} \neq W_{B0},$$

and therefore

$$W_E + W_B = W_{E0} \cos^2 \omega t + W_{B0} \sin^2 \omega t. \tag{45}$$

Using the double-angle identities,

$$W_E + W_B = \frac{W_{E0} + W_{B0}}{2} + \frac{W_{E0} - W_{B0}}{2} \cos 2\omega t. \tag{46}$$

Thus, if the geometry and field amplitudes were artificially held fixed while

$$W_{E0} \neq W_{B0},$$

the electromagnetic field energy represented by these two terms alone would oscillate.

For an isolated physical structure this variation cannot occur without compensation. Energy conservation therefore requires an additional dynamical response, for example through changes in field amplitudes, geometrical deformation, or spatial energy transport.

This provides a possible physical interpretation of the distinction between optimal and non-optimal amplitudes. The condition

$$W_{E0} = W_{B0}$$

is the special case in which electric and magnetic energies can exchange directly while their sum remains constant without requiring a first-order modulation of the geometry.

A non-optimal state,

$$W_{E0} \neq W_{B0},$$

may still be possible, but the electromagnetic oscillation must then be coupled more strongly to

$$R(t), \qquad a(t),$$

or to the spatial redistribution of field energy.

4.5. The energy-transfer intervals

The energy exchange is most rapid when the electric and magnetic energies are equal.

Differentiating Eq. (27),

$$\frac{du_E}{dt} = -u_0 \omega \sin 2\omega t, \tag{47}$$

while

$$\frac{du_B}{dt} = +u_0 \omega \sin 2\omega t. \tag{48}$$

Therefore,

$$\boxed{\frac{du_E}{dt} = -\frac{du_B}{dt}.} \tag{49}$$

At the moments when

$$u_E = u_B = \frac{u_0}{2},$$

we have

$$|\sin 2\omega t| = 1,$$

and hence the magnitude of the energy-transfer rate is maximal.

Conversely, when either field energy reaches its maximum,

$$u_E = u_0, \quad u_B = 0,$$

or

$$u_E = 0, \quad u_B = u_0,$$

the instantaneous transfer rate is zero.

This yields a useful sequence through one half-period:

$$\boxed{\begin{array}{c} W_E \text{ maximum} \\ \downarrow \\ \text{maximum energy-transfer rate} \\ \downarrow \\ W_B \text{ maximum} \\ \downarrow \\ \text{maximum reverse transfer rate} \\ \downarrow \\ W_E \text{ maximum.} \end{array}}$$

(50)

This intermediate transfer phase will be examined in the next section in connection with the displacement current. The important point is that the displacement current will not be introduced as a third independent store of electromagnetic energy. Instead, it will be considered as part of the dynamical process by which the field configuration changes between predominantly electric and predominantly magnetic states.

5. Oscillating Poynting Flow

The periodic exchange between electric and magnetic field energies is accompanied by an electromagnetic energy flux described by the Poynting vector,

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}. \tag{51}$$

For the local field geometry introduced in Section 2,

$$\mathbf{E} = E_n \, \hat{\mathbf{n}}, \qquad \mathbf{B} = B_\phi \, \hat{\boldsymbol{\phi}},$$

the Poynting vector is directed predominantly along the poloidal coordinate,

$$\mathbf{S} = \frac{E_n B_\phi}{\mu_0} \hat{\boldsymbol{\theta}}. \tag{52}$$

Thus, in the present model, electromagnetic energy does not flow primarily along the major circumference of the torus. Instead, the local energy flux is associated with motion around the toroidal cross-section.

Using the temporal dependence

$$E_n(t) = E_A \cos \omega t,$$

$$B_\phi(t) = B_A \sin \omega t,$$

Eq. (52) becomes

$$\mathbf{S}(t) = \frac{E_A B_A}{\mu_0} \cos \omega t \sin \omega t \, \hat{\boldsymbol{\theta}}. \tag{53}$$

Since

$$2 \sin x \cos x = \sin 2x,$$

we obtain

$$\boxed{\mathbf{S}(t) = S_0 \sin 2\omega t \, \hat{\boldsymbol{\theta}},} \tag{54}$$

where

$$S_0 = \frac{E_A B_A}{2\mu_0}. \tag{55}$$

The Poynting flow therefore oscillates at twice the frequency of the electric and magnetic fields.

5.1. Periodic reversal of the energy flow

Equation (54) implies that the direction of energy transport changes periodically.

During one part of the cycle,

$$S_\theta > 0,$$

and the energy flows in the +θ direction. A quarter of the energy-oscillation period later,

$$S_\theta = 0,$$

and the local energy flux momentarily vanishes. During the following part of the cycle,

$$S_\theta < 0,$$

so that the energy flow reverses and proceeds along the same spatial coordinate in the opposite direction.

The local process may therefore be represented as

$$\boxed{+\hat{\boldsymbol{\theta}} \longleftrightarrow -\hat{\boldsymbol{\theta}}.} \tag{56}$$

This behaviour is qualitatively different from that of a freely propagating plane electromagnetic wave. In a propagating wave, the time-averaged Poynting vector is non-zero and defines the direction of net electromagnetic energy transport. In the present standing or stationary configuration,

$$\langle \mathbf{S} \rangle = \frac{1}{T} \int_0^T \mathbf{S}(t) \, dt = 0. \tag{57}$$

Thus,

$$\boxed{\langle \mathbf{S} \rangle = 0.} \tag{58}$$

The vanishing of the time-averaged Poynting vector is one of the central properties of the proposed stationary toroidal structure.

5.2. Relation between Poynting flow and energy exchange

The phase of the Poynting flow is directly related to the exchange between electric and magnetic energies.

For the energy-balanced case,

$$u_E = u_0 \cos^2 \omega t,$$

$$u_B = u_0 \sin^2 \omega t.$$

Their time derivatives are

$$\frac{du_E}{dt} = -u_0\omega \sin 2\omega t, \tag{59}$$

$$\frac{du_B}{dt} = +u_0\omega \sin 2\omega t. \tag{60}$$

The Poynting vector obeys the same sinusoidal phase dependence,

$$S \propto \sin 2\omega t.$$

Consequently, the Poynting flow is maximal precisely when the rate of conversion between electric and magnetic energy is maximal.

When

$$u_E = u_B = \frac{u_0}{2},$$

we have

$$|\mathbf{S}| = S_0.$$

Conversely, when one of the field energies reaches its maximum and the other vanishes,

$$u_E = u_0, \quad u_B = 0,$$

or

$$u_E = 0, \quad u_B = u_0,$$

the Poynting vector vanishes,

$$\mathbf{S} = 0.$$

The sequence is therefore

$$\boxed{W_E^{\max} \to S^{\max} \to W_B^{\max} \to -S^{\max} \to W_E^{\max}.} \tag{61}$$

This provides a direct connection between the temporal energy exchange and the internal spatial transport of energy.

5.3. Poynting flow does not represent an additional energy store

It is important to distinguish between energy density and energy flux.

The quantities

$$u_E = \frac{\varepsilon_0 E^2}{2}$$

and

$$u_B = \frac{B^2}{2\mu_0}$$

represent stored electromagnetic energy per unit volume.

The Poynting vector, by contrast, has the dimensions of power per unit area,

$$[\mathbf{S}] = \mathrm{W\,m^{-2}},$$

and describes the rate and direction at which electromagnetic energy crosses a surface.

Therefore, the oscillating Poynting flow should not be interpreted as a third independent energy component. Instead, it represents the transport mechanism through which field energy is redistributed within the structure.

This distinction is expressed by Poynting's theorem,

$$\boxed{\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E},} \tag{62}$$

where

$$u = u_E + u_B.$$

In a source-free region,

$$\mathbf{J} = 0,$$

and Eq. (62) reduces to

$$\boxed{\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = 0.} \tag{63}$$

A local decrease of electromagnetic energy density must therefore be accompanied by an outward energy flux, while a local increase requires a converging Poynting flow.

5.4. Local oscillation versus global confinement

The condition

$$\langle \mathbf{S} \rangle = 0$$

does not by itself guarantee that the toroidal structure is perfectly confined.

For a localized stationary configuration, a stronger global condition is required. If V is a volume containing the complete torus and ∂V is a sufficiently distant closed surface, then no net energy should be radiated to infinity:

$$\boxed{\left\langle \oint_{\partial V} \mathbf{S} \cdot d\mathbf{A} \right\rangle = 0.} \tag{64}$$

Ideally, for a perfectly bound periodic configuration, the instantaneous or cycle-integrated outward flux should also vanish appropriately.

The internal Poynting flow may then remain non-zero while energy is redistributed between different regions of the torus. This is analogous to a resonant electromagnetic system, where substantial local energy flows may exist even though no net energy leaves the complete system.

The proposed structure therefore requires two distinct properties:

$$\boxed{\text{non-zero internal } \mathbf{S}}$$

together with

$$\boxed{\text{zero net energy escape.}}$$

The first describes internal dynamics; the second is the condition for localization.

5.5. Effect of non-optimal field amplitudes

If the actual amplitudes are not energy balanced,

$$E_A \neq E_0, \qquad B_A \neq B_0,$$

the Poynting vector still has the form

$$\mathbf{S}(t) = \frac{E_A B_A}{2\mu_0} \sin 2\omega t \, \hat{\boldsymbol{\theta}}. \tag{65}$$

Its time average remains zero as long as the fields preserve the quarter-period phase difference.

Thus,

$$\boxed{\langle \mathbf{S} \rangle = 0}$$

does not require the optimal relation between the field amplitudes.

This is an important distinction. The absence of net translational energy transport may be a more general property of the standing toroidal oscillation, whereas the condition

$$W_{E0} = W_{B0}$$

defines the more restrictive state in which the electric and magnetic energy maxima are balanced.

A non-optimal torus can therefore, at least at the level of the local oscillatory model, retain a zero mean Poynting flow while requiring additional geometrical or energetic adjustment to preserve total energy.

5.6. Stationary structure with internal electromagnetic motion

The resulting physical picture is therefore not that of a motionless electromagnetic object. Rather, it is a structure in which energy is continuously redistributed.

The centre of the torus may remain stationary while

$$\mathbf{E}(t), \qquad \mathbf{B}(t), \qquad \mathbf{S}(t)$$

all vary periodically.

The distinction can be summarized as

$$\boxed{\text{stationary structure} \neq \text{stationary field energy.}}$$
(66)

The proposed torus is stationary only in the sense that it has no persistent translational energy flow. Internally, it is a permanently dynamic electromagnetic system.

The periodic reversal of the Poynting vector also suggests that the electromagnetic stresses acting on the toroidal geometry must vary during the same cycle. The next section therefore examines the Maxwell stress tensor and the resulting dynamic equilibrium of the torus.

6. Electromagnetic Stress and Dynamic Equilibrium

The stationary toroidal structure cannot be regarded as geometrically rigid. The electric and magnetic fields exert stresses on the field configuration itself, and these stresses vary during the electromagnetic cycle. A dynamically stable torus must therefore be understood as a structure in which expansion and contraction are periodically balanced.

The appropriate quantity for describing these stresses is the Maxwell stress tensor,

$$T_{ij} = \varepsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2}\delta_{ij} B^2 \right). \quad (67)$$

For the local field geometry assumed in this work,

$$\mathbf{E} = E_n \hat{\mathbf{n}}, \qquad \mathbf{B} = B_\phi \hat{\boldsymbol{\phi}},$$

the principal components become

$$T_{nn} = \frac{\varepsilon_0 E^2}{2} - \frac{B^2}{2\mu_0}, \quad (68)$$

$$T_{\phi\phi} = -\frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}, \quad (69)$$

and

$$T_{\theta\theta} = -\frac{\varepsilon_0 E^2}{2} - \frac{B^2}{2\mu_0}. \quad (70)$$

These three components describe different mechanical effects and show that the electromagnetic stress cannot be represented by a single scalar pressure.

6.1. Oscillating normal stress

Using

$$E(t) = E_A \cos \omega t,$$

$$B(t) = B_A \sin \omega t,$$

Eq. (68) gives

$$T_{nn}(t) = \frac{\varepsilon_0 E_A^2}{2} \cos^2 \omega t - \frac{B_A^2}{2\mu_0} \sin^2 \omega t. \quad (71)$$

For the energy-balanced case,

$$\frac{\varepsilon_0 E_0^2}{2} = \frac{B_0^2}{2\mu_0} = u_0,$$

, and therefore

$$T_{nn} = u_0 \left(\cos^2 \omega t - \sin^2 \omega t \right).$$

Thus,

$$\boxed{T_{nn}(t) = u_0 \cos 2\omega t.} \quad (72)$$

The normal stress therefore reverses direction twice during one field period.

When the electric field reaches its maximum,

$$E = E_0, \qquad B = 0,$$

we obtain

$$T_{nn} = +u_0.$$

When the magnetic field reaches its maximum,

$$E = 0, \qquad B = B_0,$$

the sign is reversed,

$$T_{nn} = -u_0.$$

The local surface of the torus is therefore alternately subjected to outward and inward electromagnetic stress.

This gives a direct mechanism for periodic geometrical deformation.

6.2. Toroidal magnetic tension

The component along the major circumference is

$$T_{\phi\phi} = -T_{nn}. \tag{73}$$

The magnetic part of this stress is particularly important because the magnetic field lines are curved around the torus.

A magnetic field possesses tension along its field lines of characteristic magnitude

$$\tau_B = \frac{B^2}{2\mu_0}. \tag{74}$$

For a straight field line this tension does not create a net transverse force. For a field line curved with radius R, however, the tension produces an inward force associated with the curvature.

Its characteristic force density is of order

$$\boxed{f_{\mathrm{curv}} \sim \frac{B^2}{\mu_0 R}.} \tag{75}$$

The exact coefficient depends on the spatial distribution of the field, but the geometrical dependence is essential: a smaller major radius implies stronger curvature and therefore a stronger magnetic contraction effect.

This provides a natural mechanism opposing the electric tendency to expansion.

The competition may be represented schematically as

$$\boxed{\text{electric stress} \longleftrightarrow \text{magnetic curvature tension.}}$$

6.3. Stress in the poloidal direction

The third principal component is

$$T_{\theta\theta} = -\left(\frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}\right). \tag{77}$$

For the energy-balanced oscillation,

$$u_E + u_B = u_0,$$

and therefore

$$\boxed{T_{\theta\theta} = -u_0.} \tag{78}$$

Unlike Tnn, this component does not oscillate in the idealized balanced case.

This is an important result. The electromagnetic torus contains simultaneously:

$$\boxed{\text{an oscillating normal stress}}$$

and

$$\boxed{\text{a persistent poloidal tension.}}$$

The geometry can therefore possess a continuously acting confining component while another component periodically changes direction.

6.4. Dynamic rather than static equilibrium

It would be too restrictive to require the electromagnetic forces to cancel at every instant.

Indeed, Eq. (72) shows that even in the ideal energy-balanced state,

$$T_{nn} \neq 0$$

during most of the cycle.

The appropriate condition is therefore not

$$F_E(t) = F_B(t)$$

at all times, but a periodic dynamic equilibrium in which the torus repeatedly expands and contracts without secular growth or collapse.

Let q denote any geometrical coordinate of the torus. Its motion can schematically be written as

$$M_q \ddot{q} = F_q(t). \tag{79}$$

A stable periodic structure requires

$$q(t + T_q) = q(t), \tag{80}$$

rather than

$$\dot{q} = \ddot{q} = 0.$$

Thus, the equilibrium state of the torus is better described as a **limit cycle** or periodic dynamical state than as a static force balance.

6.5. Characteristic frequency of the mechanical forcing

Since

$$T_{nn} \propto \cos 2\omega t,$$

the principal oscillatory electromagnetic force acts with angular frequency

$$\boxed{\Omega_F = 2\omega.} \tag{81}$$

Therefore, the geometrical response is naturally expected to contain a component at the same frequency,

$$R(t), a(t) \sim \cos(2\omega t + \phi),$$

where the phase ϕ depends on the effective electromagnetic inertia and restoring forces of the torus.

The geometrical oscillation need not be exactly in phase with the electromagnetic stress. In fact, the displacement of an oscillating system is generally phase shifted relative to the applied force.

This distinction is important when comparing the extrema of the field strength with the extrema of the toroidal dimensions.

The moment of maximum electric stress is not necessarily the moment of maximum radius. It may instead correspond to maximum acceleration of the geometry.

6.6. Two-dimensional geometrical response

Because the torus has two characteristic dimensions,

$$R(t), \qquad a(t),$$

the electromagnetic stress produces at least two coupled geometrical responses.

For small deviations,

$$x(t) = R(t) - R_0,$$

$$y(t) = a(t) - a_0,$$

the most general linearized form is

$$M_R \ddot{x} + K_{RR} x + K_{Ra} y = F_R^{(0)} \cos 2\omega t, \tag{82}$$

$$M_a \ddot{y} + K_{aR} x + K_{aa} y = F_a^{(0)} \cos 2\omega t. \tag{83}$$

The coefficients Kij describe how a change in one dimension modifies the electromagnetic stress acting on either dimension.

The off-diagonal terms,

$$K_{Ra}, \qquad K_{aR},$$

are especially important because the two radii are not independent electromagnetically. A change in R changes the curvature of the magnetic field, while a change in a changes the spatial confinement and field amplitudes.

The stationary torus is therefore fundamentally a **coupled electromagnetic-geometrical oscillator**.

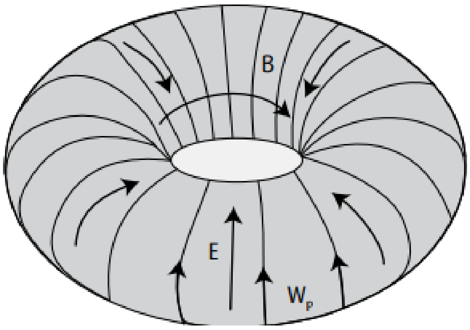

Figure 2. Schematic field geometry of the stationary electromagnetic torus.
The magnetic field Bϕ is assumed to follow the toroidal direction along the major circumference of the torus, whereas the electric field is predominantly transverse and locally normal to the toroidal surface. The figure is schematic and indicates only the principal field orientations; the exact spatial distributions required by Maxwell's equations and by the condition of zero net electric flux remain to be determined.

6.7. Stability condition

For the unforced linearized system,

$$M\ddot{\mathbf{q}} + K\mathbf{q} = 0,$$

small oscillations are stable only if the corresponding squared eigenfrequencies are positive.

In the simplest case this requires

$$K_{RR} > 0, \qquad K_{aa} > 0,$$

and

$$\boxed{K_{RR}K_{aa} - K_{Ra}K_{aR} > 0.} \tag{84}$$

These conditions express a physically clear requirement: if the torus is slightly displaced away from its equilibrium geometry, the resulting electromagnetic stresses must act so as to return it toward that geometry.

Thus,

$$R > R_0$$

must on average produce a contraction tendency, whereas

$$R < R_0$$

must produce an expansion tendency.

The analogous condition applies to the minor radius a.

Whether a particular toroidal field distribution satisfies Eq. (84) is a nontrivial question and ultimately requires the complete spatial functions

$$\mathbf{E}(\mathbf{r}, t), \qquad \mathbf{B}(\mathbf{r}, t).$$

The present analysis establishes the structure of the stability problem but does not yet provide a unique global toroidal eigenmode.

6.8. Role of the energy-optimal amplitudes

For arbitrary actual amplitudes,

$$E_A, \qquad B_A,$$

the cycle-averaged normal stress is

$$\langle T_{nn} \rangle = \frac{1}{4}\left(\varepsilon_0 E_A^2 - \frac{B_A^2}{\mu_0}\right). \tag{85}$$

The energy-balanced relation

$$E_0 = cB_0$$

therefore gives

$$\boxed{\langle T_{nn} \rangle = 0.} \tag{86}$$

This gives the optimal amplitudes an additional physical meaning: they are not only associated with equal maximum electric and magnetic energies, but also with vanishing mean normal electromagnetic stress in the local symmetric model.

For

$$E_A > cB_A,$$

the mean normal stress is biased toward the electric contribution.

For

$$E_A < cB_A,$$

the magnetic contribution dominates.

A non-optimal torus may therefore shift its mean dimensions until geometrical changes restore a dynamically balanced configuration.

This reinforces the distinction introduced earlier:

energy-optimal amplitudes ≠ necessary amplitudes for existence. (87)

The optimal state may represent the centre of the dynamical stability region rather than its only permitted point.

6.9. Physical picture of the equilibrium cycle

The complete local cycle can now be summarized.

At maximum electric field,

$$E = E_0, \qquad B = 0,$$

the normal electromagnetic stress is maximally biased toward expansion.

As the electric field decreases and the magnetic field grows, energy flows internally and the normal stress decreases.

When

$$u_E = u_B,$$

the normal stress passes through zero while the Poynting flow is maximal.

At maximum magnetic field,

$$E = 0, \qquad B = B_0,$$

the normal stress has reversed and magnetic confinement is strongest.

The sequence then reverses.

Thus,

$$\boxed{\begin{array}{c}\text{electric domination}\\ \downarrow\\ \text{maximum internal energy flow}\\ \downarrow\\ \text{magnetic domination}\\ \downarrow\\ \text{reverse energy flow}\\ \downarrow\\ \text{electric domination}\end{array}} \tag{88}$$

is simultaneously an **energy cycle** and a **mechanical stress cycle**.

The stationary torus is therefore not maintained by a static cancellation of forces. Its proposed confinement is a dynamical process in which electromagnetic energy, momentum flux, and geometry continuously respond to one another.

The following section examines the resulting oscillation of the two toroidal dimensions R(t) and a(t) in greater detail.

7. Oscillation of the Toroidal Geometry

The electromagnetic stresses derived in the previous section imply that the stationary torus should not be expected to maintain a perfectly fixed geometry. In the energy-balanced state, the normal Maxwell stress oscillates as

$$T_{nn}(t) = u_0 \cos 2\omega t. \tag{89}$$

The electromagnetic fields therefore exert alternating expansion and contraction stresses on the toroidal structure. Since these stresses vary at twice the field frequency, the geometrical response is also expected to contain a component with angular frequency 2ω.

The torus is described by two characteristic dimensions: the major radius R and the minor radius a. Their time dependence can be written generally as

$$R = R(t), \qquad a = a(t). \tag{90}$$

The resulting motion should be interpreted as an internal deformation of the electromagnetic structure rather than translational motion of the torus as a whole.

7.1. Small oscillations around an equilibrium geometry

Let

$$R_0, \qquad a_0$$

denote the mean dimensions of a dynamically balanced torus. Small deviations are defined by

$$\begin{aligned} x(t) &= R(t) - R_0, \\ y(t) &= a(t) - a_0. \end{aligned} \tag{91}$$

The simplest periodic representation is

$$R(t) = R_0 \left[1 + \epsilon_R \cos(2\omega t + \phi_R)\right], \tag{92}$$

and

$$a(t) = a_0 \left[1 + \epsilon_a \cos(2\omega t + \phi_a)\right], \tag{93}$$

where

$$\epsilon_R \ll 1, \qquad \epsilon_a \ll 1$$

in the small-deformation approximation.

The phases ϕ_R and ϕ_a need not coincide with the phase of the electromagnetic stress. The force determines acceleration rather than displacement, and the resulting phase relation depends on the effective inertia and restoring properties of the electromagnetic structure.

Thus, maximum electric-field stress does not necessarily coincide with the maximum geometrical size of the torus.

7.2. Why the geometrical frequency is 2ω

The doubling of frequency follows directly from the quadratic dependence of electromagnetic stress on field strength.

Since

$$E \propto \cos \omega t,$$

the electric contribution to stress behaves as

$$E^2 \propto \cos^2 \omega t.$$

Similarly,

$$B^2 \propto \sin^2 \omega t.$$

Their difference contains

$$\cos^2 \omega t - \sin^2 \omega t = \cos 2\omega t.$$

Thus,

$$\boxed{\Omega_{\text{geom}} = 2\omega} \tag{94}$$

for the directly forced geometrical component.

During one complete period of the electric field,

$$T = \frac{2\pi}{\omega},$$

the geometry therefore undergoes two deformation cycles,

$$T_{\text{geom}} = \frac{\pi}{\omega} = \frac{T}{2}. \tag{95}$$

This has an intuitive interpretation. Reversing the direction of the electric field,

$$E \to -E,$$

does not reverse the electric stress because the stress depends on E2E^2E2. The same is true for the magnetic field. Geometry therefore responds to field intensity rather than field polarity.

7.3. Two coupled geometrical modes

Because R and a represent independent geometrical degrees of freedom, several types of deformation are possible.

In the first mode, both radii vary approximately in the same phase:

$$\delta R > 0, \qquad \delta a > 0.$$

The entire torus expands, and half a geometrical period later both dimensions contract. This may be called a **global breathing mode**.

In the second mode,

$$\delta R > 0, \qquad \delta a < 0,$$

and subsequently

$$\delta R < 0, \qquad \delta a > 0.$$

The torus then alternates between a larger and thinner configuration and a smaller and thicker configuration.

This may be called a **shape-oscillation mode**.

The distinction is important because these two modes have very different effects on the volume and therefore on the electromagnetic field amplitudes.

7.4. Volume variation

For an ideal torus,

$$V = 2\pi^2 R a^2. \tag{96}$$

For small changes,

$$dV = 2\pi^2 \left(a^2\, dR + 2Ra\, da\right).$$

Dividing by V gives

$$\boxed{\frac{\delta V}{V} = \frac{\delta R}{R} + 2\frac{\delta a}{a}.} \tag{97}$$

If R and a oscillate in phase, the corresponding changes reinforce each other and the volume oscillation can be substantial.

However, if they oscillate in opposite phase, their contributions may partly or almost completely cancel.

A particularly interesting limit is

$$\boxed{\frac{\delta R}{R} + 2\frac{\delta a}{a} \simeq 0.} \tag{98}$$

In this case,

$$\delta V \simeq 0$$

to first order.

Equation (98) gives

$$\boxed{\frac{\delta a}{a} \simeq -\frac{1}{2}\frac{\delta R}{R}.} \tag{99}$$

Thus, a one-percent increase of the major radius would be accompanied by approximately a half-percent decrease of the minor radius.

For example,

$$R(t) = R_0 \left[1 + \epsilon \cos 2\omega t\right] \tag{100}$$

would correspond approximately to

$$a(t) = a_0 \left[1 - \frac{\epsilon}{2} \cos 2\omega t\right]. \tag{101}$$

The torus then continuously transforms as

$$\boxed{\text{large and thin} \longleftrightarrow \text{small and thick.}} \tag{102}$$

This deformation is particularly attractive for an electromagnetic structure because it allows substantial geometrical motion without requiring a comparably large variation of the total occupied volume.

7.5. Coupling between geometry and field amplitudes

Changes in the toroidal dimensions affect the field amplitudes because the same electromagnetic energy is distributed over a different volume.

For approximately fixed spatial profiles, the field energy scales as

$$W \sim Ra^2 \left(\varepsilon_0 E_A^2 + \frac{B_A^2}{\mu_0}\right). \tag{103}$$

If the total energy and the relative electric-to-magnetic distribution remain approximately fixed over a slow geometrical deformation, then

$$E_A, B_A \propto (Ra^2)^{-1/2}. \tag{104}$$

Relative to the equilibrium geometry,

$$\boxed{E_A(R,a) \simeq E_0 \left(\frac{R_0 a_0^2}{Ra^2}\right)^{1/2},} \tag{105}$$

and

$$\boxed{B_A(R,a) \simeq B_0 \left(\frac{R_0 a_0^2}{Ra^2}\right)^{1/2}.} \tag{106}$$

For the approximately volume-preserving mode of Eq. (98),

$$Ra^2 \simeq R_0 a_0^2,$$

and therefore

$$\boxed{E_A \simeq E_0, \qquad B_A \simeq B_0} \tag{107}$$

to first order.

This means that the toroidal geometry may oscillate while the electromagnetic amplitudes remain relatively stable.

By contrast, a global breathing mode in which R and a increase together produces a reduction of both field amplitudes, while contraction produces their increase.

7.6. Geometry and oscillation frequency

The dimensions of the torus also determine the spatial scales of the electromagnetic mode.

A general resonant condition may be written as

$$\omega = \Gamma \frac{c}{L_{\mathrm{eff}}(R,a)}, \tag{108}$$

where Γ is a dimensionless geometrical eigenvalue and L_{eff} is an effective characteristic length.

Consequently,

$$\frac{\delta\omega}{\omega} = -\frac{\delta L_{\mathrm{eff}}}{L_{\mathrm{eff}}}. \tag{109}$$

If the characteristic spatial variation is governed predominantly by the minor radius,

$$L_{\mathrm{eff}} \sim a,$$

then

$$\boxed{\frac{\delta\omega}{\omega} \simeq -\frac{\delta a}{a}.} \tag{110}$$

Thus, a thinner torus would tend to support a higher electromagnetic oscillation frequency, whereas a thicker torus would support a lower frequency.

This introduces an additional feedback mechanism:

$$a \to \omega \to E, B \to T_{ij} \to a. \tag{111}$$

The field and geometry should therefore ultimately be solved as one coupled dynamical system rather than treated independently.

7.7. Effective inertia of the geometry

A changing electromagnetic geometry possesses inertia because electromagnetic energy carries momentum and contributes to the energy-momentum tensor.

For an order-of-magnitude description, an effective inertial scale may be written as

$$M_{\text{eff}} \sim \frac{W}{c^2}. \tag{112}$$

For each deformation mode it is more appropriate to introduce separate effective inertial coefficients,

$$M_R = \alpha_R \frac{W}{c^2},$$
$$M_a = \alpha_a \frac{W}{c^2}, \tag{113}$$

where α_R and α_a depend on the detailed field geometry.

The small oscillations may then be represented by the coupled equations

$$M_R\ddot{x} + K_{RR}x + K_{Ra}y = F_R^{(0)} \cos 2\omega t, \tag{114}$$

$$M_a\ddot{y} + K_{aR}x + K_{aa}y = F_a^{(0)} \cos 2\omega t. \tag{115}$$

These equations show explicitly that the field oscillation at ω acts as a periodic driver of the geometry at 2ω.

The resulting amplitudes and phases depend on the electromagnetic stiffness matrix K and cannot be uniquely determined without a complete spatial field solution.

7.8. Periodic rather than fixed size

A central consequence of the model is therefore that the concept of a unique instantaneous radius is inappropriate.

Even in the energy-optimal state,

$$R(t) \neq R_0$$

and

$$a(t) \neq a_0$$

during most of the cycle.

Instead,

$$R_0 = \langle R(t) \rangle,$$
$$a_0 = \langle a(t) \rangle$$

represent mean dimensions.

The physical torus repeatedly moves through a range

$$R_{\min} \leq R(t) \leq R_{\max},$$

and

$$a_{\min} \leq a(t) \leq a_{\max}. \tag{116}$$

A stable torus therefore does not require a permanently fixed size. It requires that these bounds remain finite and repeat periodically:

$$\boxed{R(t + T_{\text{geom}}) = R(t),}$$
$$\boxed{a(t + T_{\text{geom}}) = a(t).} \tag{117}$$

This provides a more appropriate definition of geometrical stability for the proposed stationary electromagnetic structure.

7.9. Dynamic self-adjustment

The coupling between field amplitudes and geometry also suggests how a torus could respond to a departure from the energy-optimal state.

Suppose that the electric contribution becomes temporarily too strong. The resulting change in Maxwell stress tends to deform the torus. This deformation changes R and a, which changes the field amplitudes and possibly the resonant frequency. The altered fields then modify the electromagnetic stress.

The feedback loop may be written schematically as

$$\boxed{E/B \to T_{ij} \to R, a \to E, B, \omega \to T_{ij}.} \tag{118}$$

A stable stationary torus would correspond to a periodic solution of this feedback cycle rather than to a single static point.

This provides a possible basis for understanding why the energy-optimal amplitudes need not represent the only amplitudes at which the structure can oscillate. A non-optimal field state may correspond to a different instantaneous or mean geometry.

The distinction between **optimal amplitudes** and **permitted amplitudes** is examined more explicitly in the following section.

8. Optimal and Non-Optimal Field Amplitudes

The analysis developed so far identifies a particularly simple state in which the maximum electric and magnetic field energies are equal. This state will be referred to as the **energy-optimal configuration**. It is important, however, not to interpret this condition as a requirement that every stationary toroidal oscillation must have exactly the same electric-to-magnetic field ratio.

The distinction between an optimal state and an allowed state is central to the present model.

Let the actual field amplitudes of a particular torus be

$$E_A, \qquad B_A. \tag{119}$$

The corresponding maximum electric and magnetic energy densities are

$$u_{EA} = \frac{\varepsilon_0 E_A^2}{2}, \tag{120}$$

and

$$u_{BA} = \frac{B_A^2}{2\mu_0}. \tag{121}$$

For the simplest locally symmetric configuration, the energy-optimal amplitudes E_0 and B_0 satisfy

$$\boxed{\varepsilon_0 E_0^2 = \frac{B_0^2}{\mu_0},} \tag{122}$$

or equivalently,

$$\boxed{E_0 = cB_0.} \tag{123}$$

For a complete toroidal field distribution, the more general condition is equality of the integrated maximum field energies,

$$\boxed{\frac{\varepsilon_0}{2}\int E_0^2(\mathbf{r})\,dV = \frac{1}{2\mu_0}\int B_0^2(\mathbf{r})\,dV.} \tag{124}$$

Equation (124) should therefore be regarded as the fundamental definition of the energy-optimal state.

8.1. Why the optimal state is special

For the idealized oscillation

$$E(t) = E_0 \cos\omega t,$$

$$B(t) = B_0 \sin\omega t,$$

the electric and magnetic energies become

$$W_E = W_0 \cos^2\omega t,$$

$$W_B = W_0 \sin^2\omega t.$$

Their sum is then

$$\boxed{W_E + W_B = W_0.} \tag{125}$$

Thus, in the optimal state, the periodic exchange between electric and magnetic energy can occur without requiring a modulation of their total electromagnetic energy in the idealized fixed-geometry approximation.

The same condition also gives, locally,

$$\langle T_{nn}\rangle = 0. \tag{126}$$

The optimal state therefore has two related properties: equal maximum electric and magnetic energies and zero mean normal

Maxwell stress in the simplified symmetric geometry.

This provides a physical reason for describing the state as optimal rather than merely defining it algebraically.

8.2. Parameterization of non-optimal states

To describe deviations from the optimal amplitudes, let

$$E_A = \alpha E_0,$$
$$B_A = \beta B_0, \tag{127}$$

where α and β are dimensionless parameters.

The ratio

$$q = \frac{E_A}{cB_A} \tag{128}$$

then becomes

$$\boxed{q = \frac{\alpha}{\beta}.} \tag{129}$$

The optimal state corresponds to

$$q = 1.$$

Electric-dominated configurations have

$$q > 1,$$

whereas magnetic-dominated configurations have

$$q < 1.$$

This single parameter is useful for describing how far a particular toroidal configuration lies from the locally energy-balanced state.

8.3. Energy behaviour away from the optimum

For arbitrary amplitudes,

$$W_E(t) = W_{E0}^{(A)} \cos^2 \omega t,$$
$$W_B(t) = W_{B0}^{(A)} \sin^2 \omega t, \tag{130}$$

where

$$W_{E0}^{(A)} \propto E_A^2, \qquad W_{B0}^{(A)} \propto B_A^2.$$

Their sum is

$$W_{\mathrm{EM}}(t) = \frac{W_{E0}^{(A)} + W_{B0}^{(A)}}{2} + \frac{W_{E0}^{(A)} - W_{B0}^{(A)}}{2} \cos 2\omega t. \tag{131}$$

Only when

$$W_{E0}^{(A)} = W_{B0}^{(A)}$$

does the second term vanish.

For a non-optimal configuration, the simple two-component fixed-geometry description therefore predicts a periodic variation of W_{EM}. An isolated torus cannot obtain or lose energy without an associated energy flux or change in another degree of freedom.

Consequently, Eq. (131) should not be interpreted as a violation of energy conservation. Instead, it shows that a non-optimal torus cannot in general be described by fixed amplitudes and fixed geometry simultaneously.

At least one of the quantities

$$E_A, \quad B_A, \quad R, \quad a, \quad \omega$$

must respond dynamically.

8.4. Mean electromagnetic stress away from the optimum

The cycle-averaged normal Maxwell stress is

$$\langle T_{nn} \rangle = \frac{1}{4} \left(\varepsilon_0 E_A^2 - \frac{B_A^2}{\mu_0} \right). \tag{132}$$

Using Eq. (127) and the optimal relation $E_0 = cB_0$

,

$$\boxed{\langle T_{nn}\rangle = \frac{\varepsilon_0 E_0^2}{4}(\alpha^2 - \beta^2).} \tag{133}$$

Thus, for

$$\alpha > \beta,$$

the time-averaged stress is biased toward the electric contribution, whereas for

$$\beta > \alpha,$$

it is biased toward the magnetic contribution.

The toroidal geometry can therefore respond systematically to the field imbalance rather than merely oscillating symmetrically around its previous dimensions.

Schematically,

$$q > 1 \quad \Rightarrow \quad \text{electric-dominated deformation,}$$

whereas

$$q < 1 \quad \Rightarrow \quad \text{magnetic-dominated deformation.} \tag{134}$$

The precise change in R and a depends on the complete spatial field distribution.

8.5. Geometrical compensation

Suppose that the actual amplitudes depart slightly from their optimal ratio. The resulting non-zero mean stress changes the mean geometry from

$$(R_0, a_0)$$

to

$$(R_{\mathrm{eq}}, a_{\mathrm{eq}}).$$

The changed geometry then modifies the electromagnetic mode itself. In general,

$$E_A = E_A(R,a),$$
$$B_A = B_A(R,a),$$

and

$$\omega = \omega(R,a). \tag{135}$$

A new dynamically balanced configuration may therefore satisfy a condition of the form

$$\boxed{\langle F_R(E_A, B_A, R, a)\rangle = 0,} \tag{136}$$

$$\boxed{\langle F_a(E_A, B_A, R, a)\rangle = 0.} \tag{137}$$

Importantly, Eqs. (136) and (137) do not necessarily require

$$E_A = cB_A.$$

The geometrical factors entering the two electromagnetic stresses may compensate for a non-optimal amplitude ratio.

This leads to the possibility of a family of stationary oscillating configurations rather than a single uniquely permitted amplitude.

8.6. Amplitude self-adjustment

The feedback process can be represented as

$$\boxed{\frac{E_A}{cB_A} \to T_{ij} \to R, a \to E_A, B_A, \omega \to T_{ij}.} \tag{138}$$

If this feedback is restoring, a small deviation from the energy-optimal state can be compensated by a change in geometry.

In this sense, the amplitudes may be said to **self-adjust through the geometry**.

The statement should not be understood as implying that an arbitrary pair E_A, B_A necessarily produces a stable torus. Stability requires that the feedback have the correct sign. A perturbation away from equilibrium must generate a response that tends to reduce rather than amplify the perturbation.

Mathematically, this corresponds to the positive-stability conditions for the coupled system discussed in Section 6.

8.7. Optimal amplitude versus optimal geometry

There are consequently two related but distinct concepts:

$$(E_0, B_0)$$

describes an optimal field balance, whereas

$$(R_0, a_0)$$

describes an optimal mean geometry.

These quantities are coupled but need not be independent universal constants.

For a given total energy W, the optimal state may be represented schematically as

$$\boxed{W \longrightarrow (E_0, B_0, R_0, a_0, \omega_0).} \tag{139}$$

A different total energy could, in principle, correspond to another member of the same family of toroidal solutions.

This observation is important because classical Maxwell theory contains no intrinsic absolute length scale. The present analysis therefore does not yet establish that a unique R_0, a_0, or W must exist.

Any claim that the stationary torus has a unique size or energy would require an additional physical condition beyond the approximate analysis developed here.

8.8. A range of permitted amplitudes

The distinction between optimal and non-optimal amplitudes suggests a useful conceptual picture.

Instead of assuming

$$E_A = cB_A$$

as a strict existence condition, one may consider a stability region

$$q_{\min} < \frac{E_A}{cB_A} < q_{\max}, \tag{140}$$

within which periodic toroidal configurations may exist.

The value

$$q = 1$$

would then represent the energy-optimal centre of this region.

Outside the stability interval, electromagnetic stresses could become sufficiently unbalanced that no bounded periodic geometry exists.

At present, the limits

$$q_{\min}, \qquad q_{\max}$$

cannot be calculated without a complete field solution. Nevertheless, this formulation clearly separates two questions:

$$\boxed{\text{Can the torus oscillate?}}$$

And

At which amplitude ratio is the oscillation energetically optimal?

These are not necessarily the same question.

8.9. Consequence for the stationary torus model

The resulting picture is more general than one based on a single fixed relation E=cB.

The energy-optimal torus satisfies

$$W_{E,\max} = W_{B,\max}$$

and, in the simplest symmetric model,

$$E_0 = cB_0.$$

A non-optimal torus may instead have

$$E_A \neq cB_A,$$

while compensating through changes in

$$R, \qquad a, \qquad \omega$$

and in the detailed spatial distributions of the fields.

Thus,

$$\boxed{\text{optimal field balance} \subseteq \text{possible dynamically balanced states},} \quad 141$$

provided that stable non-optimal solutions actually exist.

This final qualification is important. The present analysis demonstrates how such compensation **could** occur, but does not prove the existence or width of a non-optimal stability region. That question requires a self-consistent global solution of the electromagnetic and geometrical dynamics.

The next section therefore turns to the constraint that every such solution, optimal or non-optimal, must satisfy: **conservation of the total energy of the stationary structure**

9. Energy Conservation of the Stationary Structure

Any stationary toroidal configuration proposed as an isolated electromagnetic structure must satisfy one condition independently of its detailed geometry: its total energy must be conserved. Internal exchange between electric and magnetic energy, oscillating Poynting flow, and periodic deformation of the torus are permitted, but none of these processes may create or destroy energy.

The appropriate starting point is Poynting's theorem,

$$\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E}, \quad (142)$$

where

$$u = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \quad (143)$$

is the electromagnetic energy density and

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \quad (144)$$

is the Poynting vector.

For the neutral elementary torus considered here, no material current is introduced. In a source-free region,

$$\mathbf{J} = 0,$$

and therefore

$$\boxed{\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = 0.} \quad (145)$$

Equation (145) expresses local energy conservation. Any decrease of electromagnetic energy in one region must be accompanied by electromagnetic energy flow into another region.

9.1. Conservation for the complete toroidal structure

Consider a fixed volume VVV large enough to contain the complete toroidal field configuration. Integrating Eq. (145) gives

$$\frac{d}{dt} \int_V u \, dV = -\oint_{\partial V} \mathbf{S} \cdot d\mathbf{A}. \quad (146)$$

Define the total electromagnetic energy as

$$\boxed{W_{\mathrm{EM}}(t) = \int_V \left(\frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right) dV.}$$

Then

$$\boxed{\frac{dW_{\mathrm{EM}}}{dt} = -\oint_{\partial V} \mathbf{S} \cdot d\mathbf{A}.} \quad (148)$$

Thus the total energy remains constant if no net Poynting flux escapes through the boundary:

$$\oint_{\partial V} \mathbf{S} \cdot d\mathbf{A} = 0. \quad (149)$$

For an exactly confined, non-radiating toroidal structure,

$$\boxed{W_{\mathrm{EM}}(t) = W_{\mathrm{tot}} = \mathrm{const.}} \tag{150}$$

This is a stronger requirement than the local condition

$$\langle \mathbf{S} \rangle = 0.$$

A locally oscillating Poynting vector may average to zero while some radiation is still emitted elsewhere. The complete stationary torus therefore requires the global condition of zero net energy escape.

9.2. Energy exchange in the optimal state

In the idealized energy-balanced configuration,

$$\begin{aligned} W_E(t) &= W_0 \cos^2 \omega t, \\ W_B(t) &= W_0 \sin^2 \omega t. \end{aligned} \tag{151}$$

Their sum is

$$W_E + W_B = W_0 \left(\cos^2 \omega t + \sin^2 \omega t\right),$$

and therefore

$$\boxed{W_E(t) + W_B(t) = W_0.} \tag{152}$$

The energy cycle is consequently

$$W_E \to W_B \to W_E,$$

without a change in the total electromagnetic energy.

At the same time,

$$S \propto \sin 2\omega t.$$

Hence the Poynting vector is largest when the redistribution of energy is fastest and zero when either W_E or W_B reaches an extremum.

The three quantities therefore have mutually consistent phases:

	W_E	W_B	S
$\omega t = 0$	W_0	0	0
$\omega t = \pi/4$	$W_0/2$	$W_0/2$	$+S_{\max}$
$\omega t = \pi/2$	0	W_0	0
$\omega t = 3\pi/4$	$W_0/2$	$W_0/2$	$-S_{\max}$
$\omega t = \pi$	W_0	0	0

(153)

The Poynting flow therefore represents the **transfer of energy**, not an additional contribution to the total stored energy.

9.3. Energy conservation during geometrical oscillation

The preceding result was obtained for a simplified fixed spatial profile. The proposed torus, however, changes its dimensions,

$$R = R(t), \qquad a = a(t).$$

Consequently, both the integration volume occupied by the strong fields and the spatial functions

$$\mathbf{E}(\mathbf{r}, t), \qquad \mathbf{B}(\mathbf{r}, t)$$

change during the cycle.

The total energy must therefore be calculated from the complete instantaneous field distribution,

$$\boxed{W_{\mathrm{tot}} = \int_{\mathbb{R}^3} \left[\frac{\varepsilon_0}{2} E^2(\mathbf{r}, t) + \frac{1}{2\mu_0} B^2(\mathbf{r}, t)\right] d^3 r.} \tag{154}$$

For a truly isolated stationary solution,

$$\boxed{\frac{dW_{\mathrm{tot}}}{dt} = 0.} \tag{155}$$

This expression is particularly important because it avoids artificially separating the energy of the electromagnetic fields from the energy associated with the deformation of the torus.

The torus considered here contains no material shell. Its changing geometry is itself a changing electromagnetic field configuration. Therefore, in a complete field description, the energy associated with geometrical motion must already be contained in the electromagnetic energy-momentum distribution.

9.4. Effective geometrical energy

In the reduced model of Sections 6 and 7, the geometrical dynamics were represented using effective inertial parameters such as

$$M_R, \qquad M_a.$$

One may then formally write an effective deformation energy,

$$W_{\rm def} = \frac{1}{2}M_R\dot{R}^2 + \frac{1}{2}M_a\dot{a}^2 + U_{\rm eff}(R,a). \tag{156}$$

In such a reduced description, conservation may be expressed schematically as

$$W_E + W_B + W_{\rm def} = {\rm const.} \tag{157}$$

However, W_{def} should not be interpreted as a new fundamental form of energy added to the Maxwell field.

Rather,

$$\boxed{\textbf{W}_{\textbf{def}}\textbf{ is an effective representation of part of the electromagnetic dynamics.}} \tag{158}$$

A complete solution of Maxwell's equations would describe all these contributions through Eq. (154) alone.

This distinction prevents double counting of energy.

9.5. Approximately volume-preserving deformation

A particularly simple geometrical mode was identified in Section 7:

$$\frac{\delta R}{R} + 2\frac{\delta a}{a} \simeq 0. \tag{159}$$

Since

$$V = 2\pi^2 R a^2,$$

Eq. (159) implies

$$\delta V \simeq 0$$

to first order.

If the spatial profiles remain approximately similar, the characteristic field amplitudes scale as

$$E_A, B_A \propto (Ra^2)^{-1/2}.$$

Hence the deformation of Eq. (159) gives, to first order,

$$E_A \simeq {\rm const}, \qquad B_A \simeq {\rm const}. \tag{160}$$

This suggests a comparatively simple mode of geometrical oscillation in which the torus alternates between

$$\text{larger and thinner}$$

and

$$\text{smaller and thicker}$$

without requiring large variations in either its volume or its field amplitudes.

This does not prove that such a mode is dynamically preferred, but it identifies a deformation compatible with relatively small first-order variations of the stored electromagnetic energy.

9.6. Energy conservation in a non-optimal state

The non-optimal case is more instructive.

For fixed amplitudes and fixed geometry, Section 8 gave

$$W_{\mathrm{EM}}(t) = \frac{W_{E0}^{(A)} + W_{B0}^{(A)}}{2} + \frac{W_{E0}^{(A)} - W_{B0}^{(A)}}{2} \cos 2\omega t. \tag{161}$$

When

$$W_{E0}^{(A)} \neq W_{B0}^{(A)},$$

the second term does not vanish.

For an isolated torus, however, Eq. (155) requires

$$W_{\mathrm{tot}} = \mathrm{const}.$$

Therefore the assumptions leading to Eq. (161) cannot all remain valid simultaneously.

A non-optimal isolated torus must respond dynamically through one or more of the quantities

$$E_A(t), \qquad B_A(t), \qquad R(t), \qquad a(t), \qquad \omega(t),$$

or through changes in the spatial field profiles.

Thus,

$$\boxed{W_{E,\max} \neq W_{B,\max}}$$

does not necessarily imply that the torus cannot exist. It implies that its dynamics cannot be represented simply by two fixed-amplitude sinusoidal fields in a fixed geometry.

This provides a more precise formulation of the proposed self-adjustment mechanism.

9.7. Energy conservation as a constraint on self-adjustment

Let the complete state of the torus be represented symbolically by

$$\mathcal{X}(t) = \{E, B, R, a, \omega\}.$$

Energy conservation imposes the constraint

$$\boxed{W[\mathcal{X}(t)] = W_{\mathrm{tot}}.} \tag{162}$$

The state may move through the multidimensional configuration space while remaining on a constant-energy surface.

The dynamics can therefore be pictured as

$$\boxed{E \to B \to \text{geometrical deformation} \to E} \tag{163}$$

with simultaneous internal redistribution through the Poynting flow.

No single component is required to remain constant. Only the total energy of the complete configuration is constrained.

This is especially relevant for non-optimal amplitudes: a reduction of one field contribution may be compensated not only by growth of the other field contribution but also by a change in the geometry and spatial distribution of both fields.

9.8. Periodicity of the complete state

For a stable stationary torus, energy conservation alone is not sufficient. The configuration must also return periodically to the same state.

For some period T_s,

$$\boxed{\mathcal{X}(t + T_s) = \mathcal{X}(t).} \tag{164}$$

Thus,

$$\begin{aligned} E(\mathbf{r}, t + T_s) &= E(\mathbf{r}, t), \\ B(\mathbf{r}, t + T_s) &= B(\mathbf{r}, t), \\ R(t + T_s) &= R(t), \\ a(t + T_s) &= a(t). \end{aligned} \tag{165}$$

The electromagnetic fields may have a basic period

$$T = \frac{2\pi}{\omega},$$

whereas quantities depending quadratically on the fields, including energy densities and electromagnetic stresses, contain the shorter period

$$\frac{T}{2}.$$

The complete state must therefore possess mutually consistent field and geometrical periodicities.

9.9. No net electromagnetic momentum transport

The stationary character also places a related constraint on momentum.

The electromagnetic momentum density is

$$\mathbf{g} = \frac{\mathbf{S}}{c^2}. \tag{166}$$

Since the internal Poynting vector reverses periodically,

$$\langle \mathbf{S} \rangle = 0$$

locally in the simplified oscillatory model, and correspondingly

$$\boxed{\langle \mathbf{g} \rangle = 0.} \tag{167}$$

For the complete torus, a stationary centre requires the integrated electromagnetic momentum not to produce persistent translation,

$$\boxed{\left\langle \int \mathbf{g}\, dV \right\rangle = 0.} \tag{168}$$

Thus the proposed stationary state requires not only zero net energy escape but also zero persistent net momentum.

Internally, however,

$$\mathbf{g}(\mathbf{r}, t) \neq 0$$

is entirely permitted.

The structure may therefore contain intense oscillating energy and momentum flows while its centre remains fixed.

9.10. Energy conservation as the central consistency condition

The stationary torus can now be characterized by three distinct requirements:

$$\boxed{\left\langle \oint_{\partial V} \mathbf{S} \cdot d\mathbf{A} \right\rangle = 0,} \tag{170}$$

and

$$\boxed{\mathcal{X}(t + T_s) = \mathcal{X}(t).} \tag{171}$$

The first expresses conservation of total energy, the second excludes secular radiative loss, and the third requires bounded periodic dynamics.

Together they provide a clearer definition of what is meant by a **stationary electromagnetic toroidal structure**.

Such a structure need not be static. Its electric and magnetic energies may exchange continuously, its Poynting vector may reverse direction, and its geometry may oscillate. Stationarity requires only that these internal processes form a closed periodic cycle without secular loss of energy or displacement of the structure as a whole.

The central question is therefore no longer whether the fields change with time. They clearly may. The more demanding question is whether a self-consistent spatial field distribution exists for which

$$\boxed{\text{field oscillation} + \text{internal energy flow} + \text{geometrical deformation}}$$

form a closed, finite-energy and non-radiating dynamical state.

This question defines the boundary between the consequences established within the present approximate model and the open problems that must be addressed in a complete theory

10. Discussion and Open Problems

The model developed in this paper describes a stationary toroidal electromagnetic structure as a dynamically oscillating system rather than as a static object. Its electric and magnetic fields vary periodically, their energies exchange, the internal Poynting flow reverses direction, and the toroidal geometry responds through periodic deformation. The centre of the structure may nevertheless remain stationary if the complete cycle carries no persistent net energy or momentum away from the torus.

Several elements of this picture are direct consequences of standard electrodynamics. A standing or resonant electromagnetic configuration can exhibit a phase difference between electric and magnetic fields. Electromagnetic energy is stored in the fields according to

$$u_E = \frac{\varepsilon_0 E^2}{2}, \qquad u_B = \frac{B^2}{2\mu_0},$$

and the Poynting vector

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

describes the local transport of this energy. Maxwell's stress tensor further shows that electric and magnetic fields exert direction-dependent stresses, which can produce both expansion and contraction tendencies in curved field configurations.

The proposed toroidal structure combines these known properties in a particular geometry. The magnetic field is taken to be predominantly toroidal, while the electric field is predominantly transverse. This geometry leads naturally to a Poynting flow around the toroidal cross-section and to magnetic tension associated with the curvature of the toroidal field lines. These features provide a possible mechanism for internal energy redistribution and geometrical oscillation.

10.1. Stationarity does not imply absence of motion

One of the central conceptual points of the model is the distinction between a stationary structure and static fields.

The proposed torus may satisfy

$$\frac{\partial \mathbf{E}}{\partial t} \neq 0, \qquad \frac{\partial \mathbf{B}}{\partial t} \neq 0,$$

while its mean spatial position remains unchanged.

Similarly,

$$\mathbf{S}(\mathbf{r}, t) \neq 0$$

does not necessarily imply translational propagation. The relevant condition is the absence of a persistent net energy flow,

$$\langle \mathbf{S} \rangle = 0$$

locally in the simplified model and, more generally,

$$\left\langle \oint_{\partial V} \mathbf{S} \cdot d\mathbf{A} \right\rangle = 0$$

for the complete structure.

The torus is therefore stationary only on the macroscopic level. Internally, it remains continuously dynamic.

10.2. Dynamic confinement

The proposed confinement mechanism is also dynamic rather than static. Electric and magnetic stresses need not cancel at each instant. Instead, they may alternate during the cycle.

In the ideal energy-balanced case,

$$T_{nn} \propto \cos 2\omega t.$$

The normal stress therefore changes sign periodically. This suggests that the geometrical dimensions

$$R(t), \qquad a(t)$$

should oscillate rather than remain constant.

A stable structure would consequently correspond to a periodic orbit in the coupled electromagnetic-geometrical system,

$$\mathcal{X}(t + T_s) = \mathcal{X}(t),$$

rather than to a static equilibrium point.

This form of stability is conceptually similar to other dynamically maintained systems, where instantaneous forces need not vanish but the complete motion remains bounded and periodic.

10.3. Optimal field amplitudes

The relation

$$E_0 = cB_0$$

appears naturally in the locally symmetric energy-balanced approximation. It implies equal maximum electric and magnetic energy densities and zero mean normal Maxwell stress.

However, the present analysis suggests that this relation should be interpreted as an **optimal energy-balance condition**, not automatically as a strict existence condition.

A non-optimal torus with

$$E_A \neq cB_A$$

may, in principle, compensate through changes in

$$R, \qquad a, \qquad \omega,$$

and in the spatial distributions of the fields.

The existence and width of such a stability region remain unknown. Determining it requires a complete self-consistent solution rather than the approximate amplitude model used here.

10.4. The unresolved localization problem

The most important unresolved problem is localization.

The analysis has shown that the assumed local field behaviour, phase relations, energy exchange, Poynting flow, and electromagnetic stresses are individually compatible with standard electromagnetic concepts. It has not shown that these ingredients form a globally self-confined solution in unbounded vacuum.

A complete stationary torus would require field functions

$$\mathbf{E}(\mathbf{r}, t), \qquad \mathbf{B}(\mathbf{r}, t)$$

that simultaneously satisfy

$$\nabla \cdot \mathbf{E} = 0,$$
$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$
$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t},$$

,

together with localization and finite total energy,

$$\int_{\mathbb{R}^3} \left(\frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right) d^3 r < \infty.$$

The fields must additionally avoid a secular radiative energy loss.

This is the central mathematical problem that remains open within the present model.

10.5. Absence of an intrinsic length scale

A related difficulty follows from the scale invariance of the source-free classical Maxwell equations. If one field configuration is a solution, geometrically rescaled configurations can often be constructed without introducing a preferred absolute length.

The approximate analysis therefore does not determine a unique value of

$$R_0$$

or

$$a_0.$$

Nor does it determine a unique total energy.

This is important if the structure is eventually to be associated with elementary matter. A physical elementary particle possesses characteristic energy and length scales, whereas the present classical electromagnetic model does not yet explain why one particular scale should be selected.

An additional constraint, nonlinear mechanism, boundary condition, quantization condition, or other physical principle may therefore be required.

10.6. Exact electric-field geometry

Another open question is the complete spatial structure of the electric field.

The elementary torus considered here is assumed to be electrically neutral,

Q=0.

Therefore,

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = 0.$$

The local picture of an electric field approximately normal to the toroidal surface must consequently include regions of opposite flux. The detailed function

$$E_n(\theta, \phi, t)$$

has not yet been derived.

A complete solution must identify a spatial electric-field pattern that satisfies both the local geometry required for the proposed stress mechanism and the global zero-flux condition.

This is likely to be one of the most important steps in constructing an explicit toroidal eigenmode.

10.7. Exact magnetic-field geometry

The assumption

$$\mathbf{B} \approx B_\phi \hat{\phi}$$

is likewise an idealization.

A strictly toroidal magnetic field generally varies across the cross-section, and a self-consistent time-dependent solution may require additional poloidal or radial components.

Such additional components could modify both

$$\mathbf{S}$$

and

$$\mathbf{T_{ij}}.$$

Therefore, the simple field geometry used in this paper should be regarded as the lowest-order conceptual model rather than a final exact configuration.

10.8. Stability against general perturbations

The present discussion considered primarily axisymmetric variations in

$$\mathbf{R(t)}$$

and

$$\mathbf{a(t)}.$$

A real field configuration could also be unstable to non-axisymmetric perturbations.

For example, the torus could develop variations of the form

$$\delta R(\phi, t),$$

$$\delta a(\phi, t),$$

or changes in the field amplitudes around the circumference.

A complete stability analysis would therefore require expansion into perturbation modes,

$$\delta \mathcal{X} \propto e^{im\phi - i\Omega t},$$

and determination of whether any mode has an exponentially growing amplitude.

True stability requires that all physically allowed perturbations remain bounded.

10.9. Radiation

A further major question is whether the oscillating structure radiates.

Time-dependent electromagnetic fields generally produce radiation, but radiation depends on the global multipole structure of the source or field configuration. A sufficiently symmetric periodic configuration may suppress some low-order radiative multipoles.

The relevant condition is not merely

$$\langle S \rangle = 0$$

inside the torus, but the far-field behaviour,

$$\lim_{r\to\infty} r^2 \langle S_r \rangle.$$

For a perfectly stationary bound structure this quantity must vanish.

Whether the proposed toroidal oscillation can satisfy this requirement remains to be demonstrated.

10.10. Role of the displacement current

The displacement current,

$$\mathbf{J}_D = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$

plays an important role in the time-dependent Maxwell coupling between electric and magnetic fields.

In the present interpretation it should not be treated as a third independent store of energy. Rather, it marks the changing electric-field configuration associated with the transfer between electric and magnetic states.

The detailed relationship among

$$\mathbf{J}_D, \qquad \nabla \times \mathbf{B}, \qquad \mathbf{S},$$

in the complete toroidal geometry remains to be derived.

This could provide a more rigorous description of the intermediate phase of the energy cycle.

10.11. Possible relation to elementary matter

The motivation for studying a stationary electromagnetic torus is broader than the existence of an unusual resonant field configuration.

If a finite-energy, stable and self-confined neutral toroidal solution were found, it could be considered as a candidate primitive electromagnetic structure from which more complex configurations might be constructed.

However, this interpretation lies beyond what has been demonstrated in the present work.

No identification with a known elementary particle is made here. In particular, the present model does not yet derive particle properties such as

$$\text{mass}, \qquad \text{spin}, \qquad \text{charge}, \qquad \text{magnetic moment},$$

or quantum statistics.

The current analysis should therefore be understood as investigating a possible **electromagnetic building block**, not as presenting a complete particle theory.

10.12. Central open problem

The results of this work reduce the main question to a more precise mathematical form.

The task is to determine whether there exists a nontrivial solution

$$\boxed{\{\mathbf{E}(\mathbf{r},t),\mathbf{B}(\mathbf{r},t)\}}$$

such that:

$$Q = 0,$$

$$W_{\mathrm{tot}} < \infty,$$

$$W_{\mathrm{tot}} = \mathrm{const},$$

$$\mathcal{X}(t+T_s) = \mathcal{X}(t),$$

$$\left\langle \oint_{\infty} \mathbf{S}\cdot d\mathbf{A} \right\rangle = 0,$$

and the spatial field structure remains localized around a toroidal region.

If such a solution exists and is stable against small perturbations, the stationary electromagnetic torus would become a genuine dynamical solution rather than only a geometrical hypothesis.

If no such solution exists within the linear vacuum Maxwell equations, that negative result would be equally important: it would identify precisely which additional physical ingredient is required for self-confinement.

The present analysis therefore does not close the question. It narrows it to a well-defined problem of **localization, stability, and self-consistency of a time-periodic toroidal electromagnetic field configuration**

11. Conclusions

This work has explored the possibility of a stationary toroidal electromagnetic structure whose mean position remains fixed while its internal electric and magnetic fields continue to oscillate.

The proposed configuration is based on a predominantly toroidal magnetic field and a predominantly transverse electric field. A local quarter-period phase relation,

$$E(t) = E_A \cos\omega t, \qquad B(t) = B_A \sin\omega t,$$

allows the electric and magnetic field energies to vary periodically as

$$W_E \propto \cos^2\omega t, \qquad W_B \propto \sin^2\omega t.$$

In the energy-balanced case, the maximum electric and magnetic energies are equal and their sum remains constant,

$$W_E + W_B = W_0.$$

The corresponding Poynting vector varies as

$$\mathbf{S} \propto \sin 2\omega t,$$

so that its direction reverses periodically and

$$\langle \mathbf{S} \rangle = 0.$$

Thus, the absence of translational propagation does not imply the absence of electromagnetic energy transport. Energy may move continuously within the structure while producing no persistent net flow in one direction.

The same field oscillation produces periodic electromagnetic stresses. In the simplified toroidal geometry, the normal Maxwell stress varies approximately as

$$T_{nn} \propto \cos 2\omega t.$$

Consequently, the torus is not expected to maintain a rigid geometry. Its major and minor radii,

$$R(t), \qquad a(t),$$

may oscillate at twice the basic electromagnetic frequency. A particularly simple deformation mode is one in which the two dimensions vary in opposite phase,

$$\frac{\delta R}{R}+2\frac{\delta a}{a}\simeq 0,$$

so that the torus alternates between a larger and thinner form and a smaller and thicker form while its volume remains approximately constant to first order.

The analysis also distinguishes between actual field amplitudes E_A, B_A and the energy-optimal amplitudes E_0 ,B_0. In the simplest symmetric case,

$$E_0 = cB_0,$$

but this relation has been interpreted as an optimal energy-balance condition rather than as a demonstrated condition for the existence of every toroidal oscillation. Non-optimal configurations may, in principle, compensate through changes in geometry, frequency, and field distribution.

The resulting picture is therefore that of a coupled electromagnetic-geometrical oscillator:

$$\boxed{E, B \longleftrightarrow W_E, W_B \longleftrightarrow \mathbf{S} \longleftrightarrow T_{ij} \longleftrightarrow R, a.}$$

The structure is stationary only in the sense that its centre does not propagate. Internally, it remains permanently dynamic.

Several elements of this model follow directly from standard electrodynamics: the exchange between electric and magnetic field energies in resonant configurations, oscillating Poynting flow, electromagnetic momentum transport, and direction-dependent Maxwell stresses. The central unresolved issue is whether these ingredients can form a complete, finite-energy, non-radiating and self-confined toroidal solution in unbounded vacuum.

A physically admissible stationary torus would require field functions

$$\mathbf{E}(\mathbf{r},t), \qquad \mathbf{B}(\mathbf{r},t)$$

that satisfy Maxwell's equations, zero net electric charge,

Q=0,

finite and conserved total energy,

$$W_{\text{tot}} < \infty, \qquad \frac{dW_{\text{tot}}}{dt} = 0,$$

periodicity,

$$\mathcal{X}(t+T_s) = \mathcal{X}(t),$$

and absence of secular radiative loss.

The present work does not prove the existence of such a global solution. It establishes instead a consistent local and dynamical framework in which such a solution can be sought.

If a stable self-confined toroidal field configuration of this type exists, it would represent a distinctive form of electromagnetic organization: a structure that is stationary as a whole while continuously exchanging energy internally and periodically changing its geometry.

Such a structure may then be considered as a possible primitive electromagnetic building block of matter. That broader interpretation, however, requires additional steps beyond the present analysis and is left for future work.